Fractal Model of Sediments – Useful Model for Calculating of Petrophysical Parameters

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Summary

There are numerous experimental works intending to find the dependence between porosity and resistivity of sediments of different kind and different age. However there is not enough application of these theoretical considerations to the interpretation of electromagnetic data and physical modeling of data. Mathematical modeling of petrophysical properties can be done using matrix (fractal) models. A fractal model containing n-series of spheres with corresponding radii and surrounded by a thin film of adsorbed water (DEL) has been used. This model is more suitable for real sedimentary rocks. Using the parameters of the fractal model several petrophysical parameters can be calculated namely: resistivity, effective and dynamic porosity, permeability, diffusions coefficient, volume of matrix and thickness of adsorbed water which characterizes elecroosmosis polarizability and decay constant.

Introduction

For many years determining the resistivity of layers was deemed a sufficient result of applied EM geophysics. However, problems intended to be solved by geophysics have become more complicated. Now electromagnetic methods are orientated to define other petrophysical parameters such as porosity, permeability, polarization parameters, oil and gas saturation etc. Therefore a new model for mathematical modeling of petrophysical parameters is required. Traditionally for many years the empirical Archie's law has been used for interpreting EM data. It allows for the estimation of the porosity of sediments if the dependence between resistivity and porosity (at least the structural index of porosity) is known. This model also does not take into account the influence of double electrical layers which are responsible for some kind of induced polarization effects that arise in the sediments due to applied electrical current. The structure of solid of pores is very complicated and is unique for each kind of sediment. It appears as though the first person who introduced the fractal model in petrophysics in 1948 was A.S.Semenov. Following this author a matrix (fractal) model has been proposed. Our model contains n-series of spheres with corresponding radii and surrounded by a thin film of adsorbed water. This model is more suitable for real sedimentary rocks and allows for the calculation of several petrophysical parameters: resistivity, effective and dynamic porosity, permeability, diffusion coefficient, volume of matrix and thickness of DEL which characterizes electroosmosis polarizability and decay constant.

Methodology

The matrix of rocks is usually presented by a mixture of natural dielectrics with high resistivity ρ_m, which can reach 10^{11} Ωm. Another component is the pore fillings which are presented by clay and water with much lower resistivity ρ_f. The resistivity of sediments ρ_{sed} depends on ρ_m and ρ_f, on the volume V_m and V_f = (1- V_m), and on the forms and distributions of dielectrics and conductors in the solid matrix. The resistivity of sediments can be calculated as (Dakhnov, 1962):

$$\rho_{sed} = P_P \cdot \rho_f \qquad [1]$$

where P_P is a parameter of porosity which shows the ratio between the resistivity of sediments and the resistivity of pore fillings. Usually matrix models are quite simple and contain a package of different geometrical figures (cubes, spheres, ellipsoids, cuboctahedra, rectangle plans etc.) evenly distributed in the solid matrix. The model introduced by A.S.Semenov includes the n series of spheres and as a matter of fact it is a fractal (Figure 1). For this model the parameter of porosity P_p in Equation [1] is equal to:

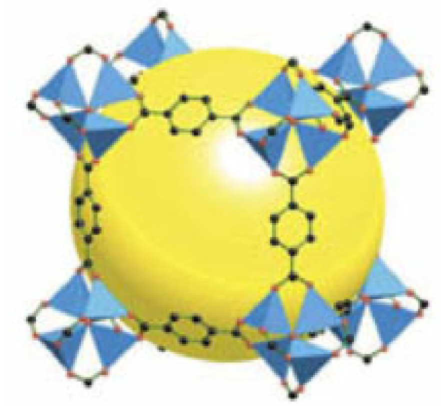

Figure 1. Fractal model.

$$P_{P\,Ps+Av} = \left[\frac{2+V_0}{2(1-V_0)}\right]^{\frac{\lg V_0}{\lg(1-V_0)}}, \qquad [2]$$

where V_0 is the coefficient of successive filling of pore space. For example, the first size fraction will occupy a volume V_0, the second size fraction, consequently, will occupy a volume $(1 - V_0) \cdot V_0$, the third size fraction $(1- V_0)^2 \cdot V_0$, etc. The remaining volume $(1- V_0)^n$ will be equal to the expected coefficient of porosity φ .
Let us consider a cube with size $L = 1$ and housed inside a sphere of radius $r_1 = L/2 = 0.5$. The volume of this sphere is equal to V_1. Let us name this sphere a grain of the first grade. Then the rest volume i.e. the porosity coefficient will be equal to $\varphi =(1 - V_1) = 0.4764$. This model presents the package of unitized spheres; it is suitable for describing well sorted sands and carboniferous bioherms. The second fraction occupies the volume $V_2=(1 - V_1) \cdot V_1 = 0.2494$. The radius of the grain of second grade is equal to $r_2 = 0.3905$. The total volume of the solid is $V_{1+2} = 0.7730$, the porosity coefficient is $\varphi = 0.227$. The volume of the third fraction is $V_3=(1 - V_1)^2 \cdot V_1 = 0.1188$. The grains of this fraction are located on the corners of the cubes of second grade, so 8 of them are housed into our cube. The radius of the sphere is $r_3 = 0.1503$, the volume of the solid is $V_{1+2+3} = 0.8918$, and the porosity coefficient is equal to 0.1082. Continuing like this we obtain the parameters of a fractal model.
However all spheres are surrounded by planes of double electric layers. If the volume of the DEL can be neglected the dynamic porosity will be equal to effective porosity. The number of fractals composing the solid can be reduced if the size of the first grade becomes smaller. In this case the volume of the double electrical layers will be more considerable. It is easy to calculate the volume of the DEL as the difference between the volume of the solid surrounded by films of adsorbed water and the volume of the pure solid. The thickness of adsorbed water τ_{aw} depends on the salinity of the electrolyte and for a salinity of 0.1 mol/l (0.56 g/l) has been accepted as 10^{-8} m (Kormiltsev, 1995). This information is very important because it is key to calculating the relation between permeability k_{PR} and effective porosity φ. Following (Kobranova, 1984) we can write:

$$k_{PR} == \frac{(1-k_{aw})^3 \cdot \varphi}{k_{aw}^2 \cdot \left(\frac{T_H}{\tau_{aw}}\right)^2 \cdot f}. \qquad [3]$$

where T_H is a tortuosity of pores (~3), f is the Kozeny constant, which ranges between 2-3, and is usually close to 2.5 and k_{aw} is the relative volume of adsorbed water. Calculating [3] for different k_{aw} we obtain the dependence φ (k_{aw}) (Figure 2). This dependence is very useful for many tasks. Obtaining information about the average maximal size of a grain (grain of the first grade) we can easily estimate the permeability of sediments if the porosity is known or calculated using geophysical data. So the next advantage of fractal modeling is the possibility to estimate the porosity of sediments via resistivity obtained by geophysical methods. The results of the interpretation of geophysical data are resistivity of unique layers or a number of layers characterized by more or less equal electrical properties. Similarly fractal model mathematical modeling of resistivity can also be done. The theoretical value of the resistivity of sediments containing three components (water, clay and solid matrix) can be calculated using Equation [4] (Kobranova, 1984).

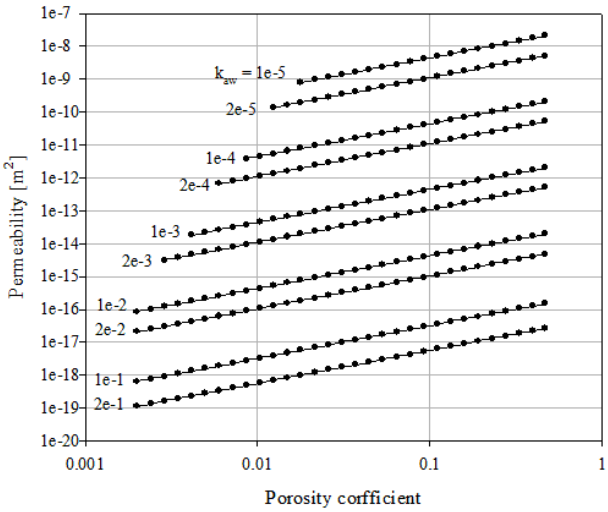

Figure 2. Permeability versus porosity for different amounts of absorbed water.

$$\rho = P_{P\,Ps+Av} \cdot \rho_{f+cl} = \frac{P_{P\,Ps+Av} \cdot \rho_f}{1 + \left(\frac{\rho_f}{\rho_{cl}} - 1\right) \cdot \frac{k_{Vclay}}{\varphi_{Ps+Av}}}. \qquad [4]$$

where P_{Ps+Av} is parameters of porosity (see (2)), ρ_f is resistivity of fluid, ρ_{cl} is resistivity of clay, φ_{Ps+Av} is porosity of solid (psammito-aleuritic fraction), k_{Vclay} is volume of clay in the unit volume of sediments. The resistivity of clay can be calculated using the follow equation:

$$\rho_{cl} = \rho_{f+cl} \cdot P_{P.cl} = \frac{P_S . P_P}{(1-\xi) \cdot \sigma_{DEL} + \xi \sigma_f}, \qquad [5]$$

where σ_f is the specific conductance of free solution, $\sigma_{DE\,L}$ is the specific conductance of double electric layers which can easily be calculated using the Poisson-Boltzman distribution, ξ is the volume of pores occupied by free electrolyte, (1 – ξ) is the volume of pores occupied by DEL, P_S is a parameter of surface conductance showing the ratio of surface conductance to specific conductance of porous clay. This parameter can be determine using calculated nomograms (see Latysheva, 1962) and can also be determined using the fractal model as part of the DEL surrounds the clay particles. P_P is a parameter of porosity of clay that can also be determined using the fractal model. However it is also possible to apply Archie's law to determine this parameter because Archie's law is intended for clayish sediments. Figure 3 shows an example of resistivity calculated for different salinities of an electrolyte. Using mathematical modeling of resistivity it is possible to estimate the porosity of sediments. Figure 4 demonstrates examples of applying fractal modeling to estimate the porosity of water bearing sediments (sands and sandstones). The investigations were carried out in the Orenburg region, Russia. The time domain method was applied and interpreted using mathematical modeling. A few layers with different resistivities were determined in the cross-section and comparison with borehole data and borehole loggings showed good agreement between log and TDEM resistivity. The water bearing sediments contained fresh water with salinity of 0.6-0.8 g/l, so using nomogram (Figure 3) the porosity of the sediments was calculated.

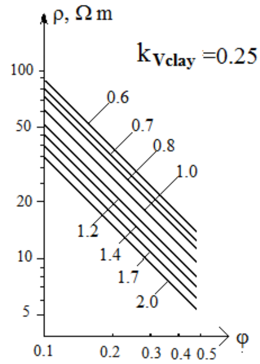

Figure 3. Calculated resistivity versus porosity coefficient of three component sediments. Index of curves is salinity of water (g/l). Volume of clay in unit is 0.25 (25%).

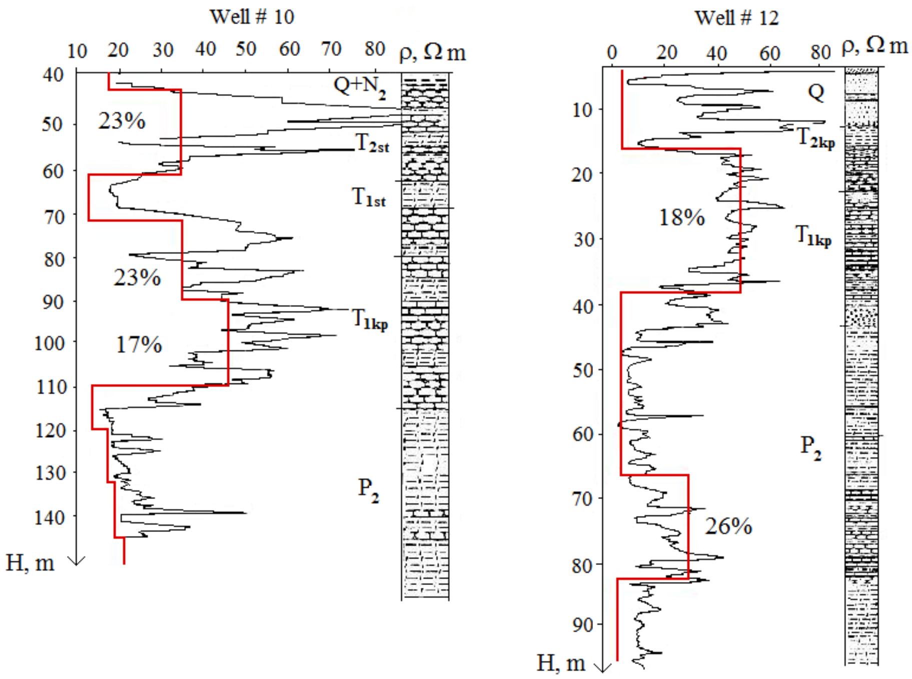

Figure 4. Borehole logs, resistivity as result of interpretation of time domain and porosity (in %) obtained using fractal model.

Conclusions

There are numerous experimental works intending to find the dependence between porosity and resistivity of sediments of different kind and different age. However there is not enough application of these theoretical considerations to the interpretation of electromagnetic data and physical modeling of data. Mathematical modeling of petrophysical properties can be done using matrix (fractal) models. A fractal model containing n-series of spheres with corresponding radii and surrounded by a thin film of adsorbed water has been used. This model is more suitable for real sedimentary rocks. Using the parameters of the fractal model several petrophysical parameters can be calculated namely: resistivity, effective and dynamic porosity, permeability, diffusions coefficient, volume of matrix and thickness of adsorbed water which characterizes elecroosmosis polarizability and decay constant. This model has been used for interpreting geophysical data for many years and for different tasks.

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