A Differential Accelerometer For Gravitational Gradiometry

N. Beverini, A. Bertolini*, A. De Michele, F. Fidecaro, F. Mango, R. Passaquieti

Dipartimento di Fisica “E. Fermi” – Università di Pisa

Summary

In this work we present the preliminary work for developing a prototype of a new class of low frequency accelerometers for geodynamics studies and space applications. The sensor is suitable to be used in a gravity gradiometer (GG) system for land geophysical survey and gravity gradient measurements. A theoretical resolution of 1 Eötvös (1 E = 10^{-9} s^{-2}) at one sample per second is achievable in a compact, lightweight (less than 2 Kg) portable instrument, operating at room temperature. The basic components of the sensor are two identical rigidly connected accelerometers separated by a 20 cm baseline vector and the useful signal is extracted as the subtraction of the two outputs, by means of an interferometric microwave readout system. The elastic structure will be engraved in a monocrystal of silicon, while the position transducer is a sapphire microwave dielectric resonator.

Introduction

The gravity gradiometric methods can avoid many of the typical problems related to the gravimetric surveys. This technique is based on the measurement of one or more of the five independent components of the tensor \( \Gamma_{ij} = \partial g_j / \partial x_i \) (that is the gradient of the gravity vector \( g \)), as the difference between the values of the \( g_i \) components across a fixed distance basis. The on-field implementation of the gradiometric principle can be made by reading, with a high sensitivity gravimetric instrument, the vertical gravity values \( g_z \) in two different point, separated by a known vectorial basis \( r_i \), and evaluating the \( \Gamma_{iz} \) component of the tensor. However, the full exploitation of the peculiarity of the gradiometric surveys could be achieved by using a single instrument with built-in differential readout capability, in order to efficiently reject common mode noise and spurious signal, like kinematical effects due to linear accelerations of the measurement platform. Our apparatus consists in two identical accelerometric sensors, placed vertically at a fixed distance \( h \), of the order of 20 cm. Each sensor (Figures 1 and 2) includes a cantilever, grinded by ultrasonic carving in a silicon monocrystal slab. The cantilever remains suspended to the frame by two thin torsion beams, and acts a torsion pendulum. The monolithic design also allows very low hysteresis, eliminating shear effects at the contact surfaces between separate mechanical parts (stick-and-slip) and ultrasonic machining yields complex shapes in crystals, ceramics and composite materials without any release of thermal and mechanical stress into the work piece. The flexure of the cantilever under gravity action is detected by a sapphire dielectric resonator transducer (SDRT) (Blair et al., 1992), consisting in a couple of sapphire disks, rigidly connected respectively to the cantilever and to the frame, axially aligned with a \( \sim 100 \mu m \) free-space separation \( d \) for standard gravity value. This apparatus acts as a microwave resonator, resonating in a Whispering-Gallery (WG) quasi-transversal magnetic mode, which is highly sensitive to the actual value of \( d \). The material to be used for the separation block and for the cylinders which support sapphire disks will be chosen in the detailed design phase of the project, in order to compensate temperature effects.
Operation Principles

The apparatus works by comparing in a differential way the resonance frequency of the two accelerometric sensors. Assuming that the two sensors are perfectly identical, any external kinematics force due to linear acceleration of the frame is cancelled as common mode and this frequency difference is proportional to the different value of the acceleration vertical component in the sensors location, and thus to the \( z \) component of the gravity gradient tensor.

Under this hypothesis of perfect identity of the sensors, it can be easily calculated the theoretical sensitivity of the apparatus. Actually, the value of the elastic frequency resonance \( f_0 \) for the prototype apparatus that we are now developing will fall in the range between 30 and 40 Hz (a lower value, up to 20 Hz, may be obtained for the final apparatus, in order to achieve a higher sensitivity). The cantilever flexure under a gravity value \( g \) is roughly given by \( z = \frac{g}{2\pi f_0} \), that is of the order of 200 \( \mu m \). With a \( \Gamma_{zz} \approx 10 E \) and a distance between the sensor of 20 cm, the differential flexure is \( 2 \cdot 10^{-14} \) m. In our WG resonator, consisting of two optical grade sapphire disks of 42 mm of diameter, and 5 mm of thickness with 100 \( \mu m \) of free space, we observed a resonance at 11.2 GHz on the \( \text{TM}_{11,1,1} \) mode, with a transfer function \( \frac{df}{dz} \approx 6 \) MHz/\( \mu m \) and a quality factor \( Q \), of the order of \( 10^5 \). This means a frequency shift between the two resonators of the order of 0.12 Hz.

This value must be compared with the noise level of the readout system. In order to reduce this level, we are implementing (Figure 3) a fully differential interferometric scheme, introduced by Blair et al. (1994), which has been demonstrated to be capable of displacement resolutions better than \( 10^{-14} \) m. A loop oscillator, including one of the gradiometer resonators as the reference cavity, a phase shifter and a low noise high-gain microwave amplifier, generates a high-stable microwave signal. A power divider splits the microwave power between the two resonators used in reflection mode. Coupling is achieved by means of two circulators. The power reflected by the
SDRTs is compared by means of a $180^\circ$ power combiner, whose output is demodulated in-phase with respect to the loop oscillator in a low-noise double-balanced mixer (DBM). The low-pass filtered output of the DBM is a voltage proportional to the sensed differential acceleration, in the low frequency band DC $-$ 20 Hz. The balanced interferometric scheme allows for carrier suppression, enhancing the resolution, especially at low frequencies. Otherwise the sensitivity would be limited by the loop oscillator phase noise and by the frequency discriminator noise. The minimum detectable gravity gradient per bandwidth unit can be evaluated by comparing the expected signal with the noise. By considering thermal noise due to the Brownian motion of the accelerometer test masses and the phase noise of the loop oscillator, we find the equation:

$$
\Gamma'(\omega) = \frac{\omega_0^2}{\pi} \left( \frac{\omega}{\frac{df}{dx}} \right)^2 \epsilon^2 S_{\phi}(\omega) + \frac{4k_B T \omega_0}{mQ_0} \left[ (\omega^2 - \omega_0^2)^2 + \left(\omega \omega_0 / Q_0\right)^2 \right]
$$

where $\omega_0 = 2\pi f_0$ is the mechanical resonant frequency, $l$ is the accelerometer baseline, $m$ is the test mass, $Q_0$ is the mechanical quality factor, $df/dx$ is the tuning coefficient, $S_{\phi}$ is the loop oscillator phase noise spectral density and $\epsilon$ is the balancing factor of the interferometric system. At room temperature typical expected parameters (Tobar et al., 1993) are $Q_0 \approx 10^4$, $S_{\phi} \approx 10^{-3}/f^2 \text{ rad}^2/\text{Hz}$ and $\epsilon \approx 0.01$, yielding $\Gamma_{\text{min}} \approx 1 \text{ E}$ for 1-sec of integration time.

**Figure 3.** Interferometric balanced detection circuit.

**Analysis of the Apparatus**

As a matter, the two accelerometers cannot be perfectly identical. If we write the equation of motion of the two sensors in the presence of an external common acceleration $a$

$$
\begin{align*}
\ddot{x}_1 + \beta_1 \dot{x}_1 + \omega_1^2 x_1 &= g_1 + a \\
\ddot{x}_2 + \beta_2 \dot{x}_2 + \omega_2^2 x_2 &= g_2 + a
\end{align*}
$$

where $g_1$ and $g_2$ are the gravity values at the sensor location, we obtain the following equations for the differential displacement $y = x_1 - x_2$ and the common mode displacement
\[ Z = X_1 + X_2; \]

\[
\begin{aligned}
\dot{y} + \beta \ \dot{y} + \gamma \ \dot{z} + \omega_0^2 \ y + \delta \ \omega_0 \ z &= \Delta g \\
\ddot{z} + \beta_1 + \beta_2 + \gamma \ \ddot{x} + \omega_0^2 \ z + \delta \ \omega_0 \ y &= 2(g + a)
\end{aligned}
\]

where \( \beta = \frac{B_1 + B_2}{2}; \gamma = \frac{B_1 - B_2}{2}; \omega_0^2 = \frac{\omega_0^2 + \omega_1^2}{2}; \delta = \omega_1 - \omega_2. \) The coupled equations can be solved, and we find for the Fourier low frequency component (\( \beta, \delta, \omega << \omega_0, \gamma << \beta \))

\[ Y(\omega) = \frac{\Delta g}{\omega_0^2} + 2\delta \left( g + A(\omega) \right) \]

A mismatch \( \delta \) between the mechanical resonance frequencies of the two accelerometers will then produce a residual sensitivity to kinematics effect with a common mode rejection rate (CMRR) given by \( 2 \delta / \omega_0. \) In order to achieve 10 E sensitivity with our geometry in presence of 1 mm/s^2 linear acceleration, a CMRR of 120 dB is required. Careful mechanical working and assembling and a fine-tuning of the inertial momentum by microweight may result in more than 60-70 dB CMRR. An additional electrostatic fine tuning of the elastic torsion constant will be achieved by applying a high-voltage on both sides of the “flap”, which will be metallized; in this way a negative stiffness effect is obtain. This trimming can be used for an automated balancing routine, with calibration given by apparatus tilting. Implementation on a mobile platform may require an inertial suspension system, which is essential in any case for the control of accelerations coming from rotational motions.

Low value of the damping factor is required to obtain high mechanical \( Q_0 \) factor. Viscous drag will avoid by placing under vacuum the apparatus. Detection of the common mode displacement can provide the feedback for active velocity damping around the mechanical resonances frequency by well-known techniques (Acernese et al., 2004). For this purpose, each sensor will be provided by an actuator consisting in a small magnet fixed on the upper face of the moving arm, driven by a coil fixed to the frame. The same actuator can be used also to block the dynamics of the cantilever when not operative or in the occasion of possible large external shock.

Thermal effect must be strictly controlled. Actually, temperature affects geometrical dimensions, elastic constants, and dielectric constant. A model has been built, that considers all this effect, and a globally compensated structure will be projected. In this way it is possible to push temperature sensitivity of the single accelerometer beyond 10^{-6}/K. In order to have 10 E of accuracy, this means to keep the temperature difference beyond 10^{-3} K. The crystalline nature of both the elastic and the dielectric element and the vacuum packing will help to achieve quite easily the result.

**Conclusions**

We have presented the gravity gradiometer that is now under development in the Department of Physics of the Pisa University. The first prototype will be tested in static conditions, but the problems related to its implementation on a mobile platform are already under study. The sensitivity should be better than 10 E/Hz^{1/2}.

Actually, the small dimension and weight of the device make it particularly suitable for automotive small dimension vehicles. In the present design only the zz component of the gravitational tensor is measured. Multisensor apparatus can be also envisaged, that would allow the determination of the whole tensor and possibly of the gyroscopic corrections.

**References**

Acernese F. et al., 2004, A local control system for the test masses of the Virgo gravitational wave detector, Astroparticle Physics 20 617-628.