Reconstruction of Geologic Bodies in depth associated with a Sedimentary Basin using Gravity and Magnetic Data

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Summary
We present a comprehensive review of the most common gravity and magnetic interpretation methods to in depth retrieval of the geometry of two types of geologic bodies associated with a sedimentary basin: 1) sedimentary basement relief; and 2) salt bodies. In reconstructing a basement topography we identify three groups of methods: the automatic, the spectral and the nonspectral methods. The reconstruction of salt bodies geometries from gravity data usually uses the interactive forward modeling, but recently gravity inversion methods have been developed to interpret this kind of geological environment.

Introduction
In a sedimentary setting, the gravity and magnetic interpretation methods can be useful and cost-effective tools for retrieving different geologic bodies geometries in depth (x-z cross-sections or full 3D region). Specifically, we will focus our attention on two bodies: 1) basement relief; and 2) salt bodies.

Reconstruction of basement relief

Different methods have been used to map the basement relief. Here, we emphasize three groups of methods: The automated depth-estimation, the spectral and the nonspectral methods.

The automatic methods
- Automated depth-estimation methods includes the Euler and Werner deconvolutions. Two geologic settings are usually considered. The first one is a weakly magnetized basement, intruded by highly magnetized rocks older than the erosive event that shaped its relief. The tops of the intrusions coincide with the basement topography, which is inferred by estimating the horizontal position and the depths of the isolated intrusive rocks. The second geologic setting is a basement relief shaped by structures, such as faults occurring after the erosion event. The basement topography is inferred by estimating the depth to the top and the horizontal position of the faults, which are approximated by a step fault model. The mathematical basis of Werner deconvolution was developed by Werner (1953) and extended by Hartman et al. (1971), Ku and Sharp (1983), Hansen and Simmonds (1993), and Ostrowski et al. (1993). The basis of Euler deconvolution was originally presented by Thompson (1982), for profile data, and by Reid et al. (1990) for gridded data, which were subsequently, improved and extended (e.g., Barbosa et al., 1999a; Mushayandebvu et al., 2001; Silva et al., 2001a; Salem and Ravat, 2003; Silva and Barbosa, 2003; Keating and Pilkington, 2004; and Mushayandebvu et al., 2004). More recently, several new Euler-based automatic and semiautomatic approaches have been developed to produce depth-to-basement estimates (e.g., Fedi, 2007; Salem et al., 2008; and Fedi et al., 2009).

The spectral methods
- This group leads to two subgroups: the statistical spectral method (Spector and Grant, 1970), designed to estimate average depths of ensembles of magnetic or gravity sources, and the subgroup that adopts inversion methods for depth-to-basement estimation by using Parker’s (1973) forward method to rapidly compute the gravity and magnetic effect of an arbitrary interface separating two homogeneous media. Parker’s (1973) formula sets a relationship between the Fourier transform of the potential-field data \( \mathbf{d} \) (gravity and magnetic data) and the sum of the Fourier transforms of the powers of the model vector \( \mathbf{p} \) describing the topography at discrete points:

\[
F \{ \mathbf{d} \} = 2\pi m \ e^{-|k| z} \sum_{n=1}^{\infty} \frac{1}{n!} F \{ \mathbf{p}^n \},
\]

(1)
where $F$ denotes Fourier transformation, $k$ the wavenumber whose components in the $(x,y,z)$ coordinate system are $(k_x, k_y, 0)$, and $m$ is a variable involving the physical property that could be a scalar or a vector. For the gravity data $m$ is a scalar $m = G$ where $G$ is the gravitational constant and the density contrast. For the magnetic data, $m$ is the vector $m = J \cdot j \cdot K \cdot \mathbf{t}$, where $J$ is the magnetization contrast, $K$ is a vector constructed from wavevector $k$ as $K = (ik_x, ik_y, k_z)$ and $j$ and $t$ are the direction cosines of the source magnetization and of the geomagnetic field, respectively. Examples of successful inversion based on Parker’s (1973) formula to estimate a topography are given in Oldenburg (1974) and Guspi (1993) for gravity data, and Pilkington and Crossley (1986), Pilkington (2006), and Caratori Tontini et al. (2008) for magnetic data. The major advantage of these inversion methods is the rapid computation of the potential-field response. Its major restriction is that the average depth of the interface [variable $z_0$ in the above equations] must be known. Because the gravity or magnetic inverse problem for depth-to-basement estimate is an ill-posed problem with non-unique and unstable solutions, these methods stabilize the solutions either by applying a low-pass filter to the data or by employing a damping parameter. Hence, they implicitly introduce the prior information that the interface is smooth.

**The non-spectral methods** - Several methods in this group approximate, in the space domain, the sedimentary pack by a set of prisms whose tops coincide with the earth’s surface and the prisms’ thicknesses represent the unknown depths to the basement. In the literature a few methods belonging to this group either do not stabilize the solution or stabilize it by imposing proximity between the solution and the initial guess. Here, we focus our attention on inversion methods that transform the ill-posed problem of estimating basement relief into a well-posed one via the Tikhonov regularization method (Tikhonov and Arsenin, 1977). Mathematically, it consists of formulating a constrained inverse problem, which is solved by minimizing an unconstrained function $\lambda(p)$ composed by: 1) the data-misfit function $\psi(p)$ defined in the data space as a norm of the difference between the observed and predicted data, and 2) the regularizing function $\phi(p)$ defined in the parameter (model) space that imposes physical or geological attributes on a solution. This problem is solved by minimizing the unconstrained objective function

$$\lambda(p) = \psi(p) + \mu \phi(p)$$

where $\mu$ is the regularization parameter that balances the effects of $\psi(p)$ and $\phi(p)$. In this case, the solution will be biased by the information introduced by the regularizing function, which must be designed to impose, on the solution, certain characteristics of the geological setting being interpreted. This procedure increases the chances that the estimated stable solution is close to the true one. Examples include Richardson and MacInnes (1989), Barbosa et al. (1997, 1999b), Gallardo-Delgado (2003), Nunes et al. (2008), and Martins et al. (2009). The major advantages of these methods are the fact that the estimated topography is not just a scaled version of the observed data and the independence on the initial guess. Conversely, its major restriction is the large processing time. As an example, we present the results obtained by two regularizing functions. The first one is named the first-order Tikhonov regularization (see Barbosa et al., 1997, and Martins et al., 2009) that imposes a smooth geometry on the solution. Figure 1a shows the noise-corrupted Bouguer anomaly (blue lines) produced by the simulated sedimentary basin with a complex simulated basement relief (Figure 1b). Figure 1c shows the estimated depths to the basement relief using the correct values of the density contrast between the sediments and the basement. The smoothing regularization yields the basement depth estimates (Figure 1c) which reconstructs very well the true depths (Figure 1b). The second regularizing function is named total variation (TV) regularization (Martins et al., 2010) that allows estimating a discontinuous basement. Figure 2a shows a simulated sedimentary basin originated from a rift development. The gain in resolution by applying the TV regularization for retrieving a fault-bounded sedimentary basin geometry can be verified by comparing Figure 2b with the estimated basement relief using smoothing regularization (Figure 2c).
Reconstruction of salt domes

Different strategies for the reconstruction of 3D (or 2D) salt bodies from gravity data have been proposed. Some methods use the interactive gravity forward modeling (e.g., Starich et al., 1994; Yarger et al., 2001; Oezsen, 2004). Gravity inversion has been used in many plays involving salt bodies or similar structures. Most methods estimate a 3D density-contrast distribution. Bear et al. (1995) minimize the $l_2$-norm of the 3D density-contrast distribution using SVD technique. Nagihara and Hall (2001) use simulated annealing to minimize the $l_2$-norm of the 3D density-contrast distribution. Krahenbuhl and Li (2006) minimize the $l_2$-norm of the first-order derivative of the 3D density-contrast distribution using a binary genetic algorithm. Krahenbuhl and Li (2009) develop a similar approach to salt imaging by combining a genetic algorithm with simulated annealing. Silva Dias et al. (2008; 2009) use an adaptive learning scheme to estimate salt bodies crossing the nil zone, which fits the data within the measurement errors, and favors compact sources closest to an skeletal outlines of the sources. Jorgensen and Kisabeth (2000) presented a joint 3D inversion of gravity, magnetic and tensor gravity fields to estimate the base of a deepwater salt deposit. Also, Routh et al. (2001) estimate the base of a salt body from the vertical component of the gravity anomaly and five independent tensor components and assuming the knowledge of the top of the salt interface. Some gravity inversion methods estimate the geometry of isolated salt bodies by approximating them by 2D bodies with polygonal cross sections (Silva and Barbosa, 2004) or 3D polyhedral bodies (Moraes and Hansen, 2001). Below, we present a test using the gravity anomaly from Galveston Island salt dome, offshore Texas (USA) using the adaptive learning scheme proposed by Silva Dias et al. (2009). This salt dome is a homogeneous body embedded in a heterogeneous sedimentary pack and crossing a nil zone. We estimated a cylindrical salt dome (Figure 3) with an associated overhang and depth to the bottom at 4 km (after Silva Dias et al., 2008).
Conclusions
We have pointed out two geologic sources in a sedimentary basin, whose geometries that can be delineated from gravity and magnetic data: 1) basement relief of a sedimentary basin; and 2) salt bodies. We have attempted to provide a review of most existing methods available in the literature aiming at the reconstruction of these geologic bodies. We have analyzed the pros and cons of each interpretation method and presented some examples illustrating the reconstruction of these bodies.

Acknowledgments
It is certainly impossible to include all references related to a comprehensive review such as this one in just a few pages. The cited papers were selected as examples of the discussed method, and the omitted papers are not intended to be considered less important. We thank EGM 2010 organizing committee for the invitation. The authors were supported in this research by fellowships from Conselho Nacional de Desenvolvimento e Tecnológico(CNPq), Brazil.

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