CSEM and MT data sensitivity analysis for 1D anisotropic inversions

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Introduction

Electrical and electromagnetic (EM) methods are used to image the subsurface resistivity distribution which depends on various factors such as lithology, pore fluid, temperature, and chemical variations. In inverse modeling, unknown parameters are estimated from measured data. We use a probabilistic approach (Tarantola 2005) for 1D inversion which includes model and data uncertainties, together with any available *a priori* information. The aim of our sensitivity study is to understand how data respond to model parameter perturbation.

Model and data

In the following sensitivity study the methodology used is described by Rovetta et al. (2008). The model is parameterized as the horizontal and vertical resistivities of a 1D layered medium with fixed values for the thicknesses of the layers. The model space $M$ contains the resistivity vector, $m = \{\rho_1, \rho_2, \ldots, \rho_L\}$, with $L$ the number of layers. The state of information on model parameters can be described by the prior probability density $\rho_m(m)$. Data space $D$ contains the CSEM acquired data, amplitude and phase of the EM field for different separations between the transmitter and the receiver at several frequencies. The actual observed response is a point in the data space represented by $d_{\text{obs}}$, containing all the data that we want to invert. The prior information on the observed data is expressed by an *a priori* probability density function, denoted by $\rho_d(d)$. It describes the uncertainty of the acquisition procedure and the data noise. The prior information on model parameters and observations can be defined by the joint probability density $\rho(m, d)$, over the space $D \times M$. If the two spaces $D$ and $M$ are independent, $\rho(m, d) = \rho_m(m) \rho_d(d)$. We introduce the homogeneous probability density

$$\mu(d, m) = \lim_{\text{dispersion} \to 0} \rho(d, m) = \mu_d(d) \mu_m(m).$$

Forward model

The information on the correlation between observable parameters and model parameters is called theoretical probability density, defined over the space $D \times M$ as $\phi(d, m) = \phi(d | m)\mu_M(m)$. The forward model is the link between model and data parameters, denoted as $d = g(m)$, with $g$ a generally non-linear vectorial function. It is used to compute, for a given set of model parameters $m \in M$, the values of the observable parameters, $d \in D$. The forward model can contain approximations, and they are described by the probability density $\phi(d | m)$, giving the modeling uncertainties.

Solution of the inverse problem

In the probabilistic framework, the solution of the inverse problem is described by an *a posteriori* probability density combining the prior information, $\rho(d, m)$, with the information obtained from the forward model, $\phi(d, m)$ and normalized by the homogeneous probability density:

$$\sigma(d, m) = \frac{k \phi(d, m) \phi(d, m)}{\mu(d, m)},$$

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where \( k \) is a normalization constant. The marginal probability in the model space is

\[
\sigma_{m}(\mathbf{m}) = \int \sigma(\mathbf{d},\mathbf{m})d\mathbf{d}.
\]

this is the solution of the inverse problem, and it depicts also the uncertainties of the inverted model parameters. We model prior uncertainties with Gaussian probabilities. In this case, the solution can be simplified to (Tarantola, 2005):

\[
\sigma_{m}(\mathbf{m}) = k \exp\left(-\frac{1}{2}\left((\mathbf{g}(\mathbf{m}) - \mathbf{d}_{\text{obs}})^{T}C_{d}^{-1}(\mathbf{g}(\mathbf{m}) - \mathbf{d}_{\text{obs}}) + \right) + (\mathbf{m} - \mathbf{m}_{\text{prior}})^{T}C_{m}^{-1}(\mathbf{m} - \mathbf{m}_{\text{prior}})\right)
\]

with \( C_{d} \) being the covariance matrix that takes into account the uncertainties due to both the measurements and the modeling, and \( C_{m} \) the covariance matrix that takes into account the uncertainties of the prior model. The solution is obtained by maximizing the posterior probability density of the model, which is equivalent to minimizing the argument of the exponential in the previous equation. We use an iterative procedure that linearizes the forward model around the current model \( \mathbf{m}_{k} \) and obtains a new model \( \mathbf{m}_{k+1} \) using the Jacobian matrix \( \mathbf{G}_{k} \) of the derivatives of the forward model equation with respect to the current model parameters:

\[
\mathbf{m}_{k+1} = \mathbf{m}_{\text{prior}} - \left[ \mathbf{G}_{k}^{T}C_{d}^{-1}\mathbf{G}_{k} + C_{m}^{-1}\right]^{-1}\mathbf{G}_{k}^{T}C_{d}^{-1}\left[ (\mathbf{g}(\mathbf{m}_{k}) - \mathbf{d}_{\text{obs}}) - \mathbf{G}_{k}\mathbf{m}_{k}\right].
\]

Given a tolerance \( \varepsilon \), the iterative algorithm stops when

\[
|m_{k,i} - m_{k,i}| < \varepsilon, \quad \forall i = 1,\ldots,M.
\]

We finally compute the a posteriori covariance matrix of the model space, \( C_{m,\text{post}} \), that describes the uncertainty of the solution

\[
C_{m,\text{post}} = \left(\mathbf{G}_{k}^{T}C_{d}^{-1}\mathbf{G}_{k} + C_{m}^{-1}\right)^{-1}.
\]

Sensitivity study

The Jacobian matrix \( \mathbf{G} (D\times M) \) of the derivatives of the forward model equation with respect to the current model parameters gives a measure of data sensitivity to the \( i^{th} \) model parameter:

\[
s_{i} = \sum_{j} |G_{j,i}|
\]

where \( G_{j,i} \) is the local linearization of the forward problem via partial derivatives on the perturbed model. High values of \( G_{j,i} \) means that a variation of model parameter \( i \) affects the data parameter \( j \) more significantly. \( s_{h} = \{s_{1}, s_{2}, \ldots, s_{M}\} \) and \( s_{v} = \{s_{1}, s_{2}, \ldots, s_{M}\} \) are the sensitivity of the data to the horizontal and vertical resistivity respectively.
**Figure 1:** From left to right: (a) 1D anisotropic layered-earth model; (b-c) inline and broadside synthetic data generated from the model on the left using the 1D CSEM-forward modeling code.

**Numerical examples**
We consider synthetic data generated from the anisotropic layered-earth model shown in Figure 1. We analyze the data sensitivity in three different test cases: the first example shows how the data sensitivity to horizontal and vertical resistivity changes as a function of depth and frequency; in the second we compare the sensitivity of an inline data sounding with respect to a broadside one, focusing on anisotropy. In the third example we compute the Jacobian matrix for 1D CSEM and MT joint inversion to study the specific behavior of CSEM and MT data sensitivity.

**Data sensitivity to anisotropic model: main features**
Figure 1 shows inline data generated from the anisotropic model at 0.25, 0.5, 0.75 and 1 Hz. We run a forward modeling code to compute the Jacobian matrix for the Ex field component, using an isotropic uniform model formed by 30 layers (layer thickness 200 m). This layered-earth model allows us to analyze the data sensitivity as a function of depth (from the mud line to 6000 m below it). In Figure 2 the Jacobian for the Ex phase at 0.25 Hz is shown. It indicates the loss of sensitivity at deeper layers. A perturbation of the model has relatively little effect at depths more than 2000 m below the mud line for the lowest frequency. This means that, in this case, the 1D inversion could recover with confidence a target at 1000 m below the mud line (Figure 1).

The matrix on the left panel of Figure 2 describes the effect of horizontal resistivity perturbation on data. Its values are lower than data derivatives on vertical resistivity (Figure 2 on the right).

**Inline and broadside data sensitivity**
We consider broadside data generated from the same model discussed in the previous example. The Jacobian matrix for the Ex phase at 0.25 Hz (Figure 3) shows the same behavior of sensitivity as before. Sensitivity decreases with depth and frequencies, but shows higher values of partial derivative on the perturbed horizontal resistivity with respect to the inline case. This means that broadside data resolve model anisotropy better than inline data. This is underlined also by the sensitivity curves depicted in Figure 4.

**CSEM-MT sensitivity comparison**
In the case of CSEM-MT 1D joint inversion we need two different forward algorithms, one for CSEM and the other for MT data. The sensitivity of the two data types will be a function of the respective forward models. In Figure 5 the Jacobian matrix for CSEM data (on the right) and that for MT data are shown. These are computed using a starting model of 50 layers down to 11000 m below the mud line (water depth is 1000 m). The MT Jacobian shows that higher periods gain depth sensitivity and that MT data are highly sensitive to the half space resistivity, whereas, as pointed out in the previous examples, CSEM data lose sensitivity with depth. This observation underlines the potential of CSEM-MT joint inversion: CSEM data recover shallow resistivity while MT data resolves deep resistivity.

**Conclusions**
The sensitivity study could drive the construction of the a priori model for 1D inversions and its a priori probability density. The anisotropy sensitivity study for in line and broadside data demonstrates the importance of the broadside illumination to solve horizontal background resistivity distribution and horizontal resistivity anomalies not only in 1D case. Moreover the joint use of CSEM and MT data, having different sensitivities, allows the extraction of a more complete resistivity image of the sub-surface.
Reference
Tarantola A., Inverse Problem Theory, 2005, SIAM.

Figure 2: The Jacobian matrix for inline data sounding: Ex phase at 0.25 Hz (horizontal axis= rx-tx separation, from 1000 m to 10000m; vertical axis= model depth, from mud line to 6000 m b.m.l). The two images represent respectively the data sensitivity to horizontal and vertical resistivity perturbation.

Figure 3: The Jacobian matrix for broadside data sounding: Ex phase at 0.25 Hz (horizontal axis= rx-tx separation, from 1000 m to 10000m; vertical axis= model depth, from mud line to 6000 m b.m.l). The two images represent respectively the data sensitivity to horizontal and vertical resistivity perturbation.

Figure 4: Inline and broadside data sensitivity curves: the dark and the light green lines are respectively the data sensitivity to horizontal resistivity for broadside and inline data.
Figure 5: The Jacobian matrix from CSEM-MT joint inversion